Political Methodology Minor Exam
Fall 2010

If you believe a question provides insufficient information to answer, explain why you think so, state some minimal additional assumptions necessary to answer, make those assumptions, and answer accordingly. There should be enough information, but this might earn partial credit. For questions that require calculations, please show your work.

The time for the exam is eight hours. This means that you have from 8 am to 5 pm on September 20th.

This exam is open note and open book. You are not allowed to collaborate.

PART I Answer all questions. The answer to questions with mathematical derivations may be handwritten though it is preferable to type these parts if time allows.

1. A researcher estimates a regression with n = 100 iid observations. The researcher comes to you with a methodological question: if she were to go out and collect 3 times as much data (which will also be iid) how much smaller would the standard errors of her regression be? What is your answer?

2. Write down the density, likelihood, log-likelihood, and derive the MLE for the parameter λ in the poisson distribution. Prove that this estimate is unbiased. Explain what it means for this MLE to be MVUE. Sketch a proof that this MLE is MVUE.

3. What does it mean for an estimator to be consistent? If you can give a formal definition, be sure to also give an explanation in words that could be understood be a colleague who has not taken a class in statistics. What must be proved for an estimator to be consistent? Full credit will require the formal definition.

4. Consider the regression model $y = X\beta + \epsilon$, with the assumption that $E(\epsilon|X) = 0$.

    - Heteroskedasticity and autocorrelation are two different ways that this assumption can be violated. Describe how each of these phenomena can arise in social-science data.
    - A researcher says that he used heteroskedasticity robust standard errors in presenting his results and in making inferences about β. Explain how the robust standard errors will differ from the usual OLS standard errors.

5. Many large corporations and government agencies administer a preemployment test in an attempt to screen job applicants. The test is supposed to measure an applicants aptitude for the job and the results are used as part of the information for making a hiring decision. Data were collected on twenty job applicants, each of whom were hired on a trial basis for six weeks. One week was spent in a training class. The remaining five

weeks were spent on the job. The participants were selected from a pool of applicants by a method that was not related to the preemployment test scores. A test was given at the end of the training period and a work performance evaluation was developed at the end of the six-week period. These two scores were combined to form an index of job performance, denoted y_i . Let X_i be the score on the preemployment test. Applicants were also classified into two racial groups, $Z_i = 1$ for minority applicants, and $Z_i = 0$ for white applicants. Regression analysis yielded the following results:

Table 1: Government Job Application Testing Models

	Model 1	Model 2
Constant	2.01	1.03
	(1.05)	(0.87)
X_i	1.31	2.36
	(0.67)	(0.54)
Z_i	-1.91	
	(0.34)	
X_iZ_i	2.00	
	(0.95)	
R^2	.66	.55

Table 2: Summary Statistics

	Mean	Min	Max.
X_i	1.47	0.28	2.51
Z_i	0.50	0.00	1.00
Y_i	4.51	1.39	8.14

(a) How many of the job applicants are white?

(b) The model implies two relationships between preemployment test score and job performance, one for whites and one for minorities. What are the slope and intercept parameters of these separate relationships? Is the standard error for these marginal effects given to you from the basic regression output?

(c) How would you test whether race modifies the relationship between preemployment test score and job performance? Can you test that hypothesis with the information provided above?

6. Often in political science analysts estimate the following type of model:

$$Conf = \beta_0 + \beta_1 Econ + \beta_2 Cont + \beta Capab + \epsilon \quad (1)$$

Here, *Conf* is the level of conflict measured by the number of casualties. *Econ* is the level of economic growth, *Cont* is a dummy variable for whether two countries are continguous, and *Capab* is a measure of military capabilities. We would like to estimate the causal effect of economic growth on conflict. What is the primary difficulty in this case? How might one augment this equation to get a causal estimate? What assumptions would you have to make?

7. One can prove that OLS estimates are unbiased. Do so now. What is different about the assumptions behind this proof when using experimental data as opposed to observational data?

8. Say you have a bivariate regression

$$Y = \beta_0 + \beta_1 X + \epsilon$$

Assume X is measured with normal random measurement error. In other words, $X = X^* + U_i$, where $U_i \sim N(0, 1)$. Sketch a proof for the effect of the measurement error on the estimate of β_1. Does this proof generalize to the multivariate case?

9. Provide an example where least squares and maximum likelihood yield different estimates. Under what conditions is this difference inconsequential?

10. On April 15th, 1912, the Titanic collided with an iceberg and sank with much loss of life. Logistic regression models were used to analyze data available for 2,201 passengers and crew: a binary dependent variable (coded 1 for survival, 0 otherwise), with predictors measuring class of travel (0 for crew, 1 for first class, 2 for second class, 3 for third class), adult/child, and gender. Table 3 summarizes this analysis, presenting maximum likelihood estimates of coefficients (with standard errors in parentheses) and empty table entries indicating whether that the corresponding variable was omitted from the respective logit model.

    - Are logistic regression estimates unbiased?
    - Based on Model 1, what proportion of those on board survived the disaster?
    - Why is it possible to calculate statistical significance here in the exact same way as for a linear regression model?
    - Describe one method for calculating a confidence interval for a predicted probability. Why is such a method necessary?
    - What is the substantive content of the null hypothesis tested by a comparison of Models 2 and 3?
    - Use a likelihood ratio test to compare Models 2 and 3.
    - How do Models 4 and Model 5 differ? That is, what is being tested in Model 5, versus Model 4 (or vice-versa)?

Table 3: Analysis of Titanic Disaster

	Model 1	Model 2	Model 3	Model 4	Model 5
Constant	-.74	2.61			1.27
	(.05)	(0.29)			(0.34)
Class		-0.33			
(0-3)		(0.05)			
1st Class			3.11	2.07	0.80
(0-1)			(0.30)	(0.35)	(0.16)
2nd Class			2.09	1.04	-0.23
(0-1)			(0.28)	(0.34)	(0.18)
3rd Class			1.33	0.26	-1.01
(0-1)			(0.25)	(0.32)	(0.15)
Crew			2.25	1.27	
(0-1)			(0.30)	(0.34)	
Adult		-1.01	-1.06	0.11	0.11
(0-1)		(0.25)	(0.24)	(0.34)	(0.34)
Male		-2.61	-2.42	-0.72	-0.72
(0-1)		(.13)	(0.14)	(0.41)	(0.41)
Adult $\times$ Male				-1.90	-1.90
				(0.43)	(0.43)

11. Provide a formal description of overdispersion in the poisson regression model (PRM). If you estimate a PRM with overdispersed data, how will this affect the model estimates? How does one test for overdispersion? What other pathologies of count data can manifest as a positive test for overdispersion? What are the two major methods of interpretation for the PRM? Under what conditions are these two methods more or less useful?

12. What is the Independence of Irrelevant Alternatives assumption? Use conditional probabilities to sketch how it might be violated. What does it imply for models of unordered choices? What models might one use to model data when IIA is violated?

13. Table 4 contains the results from a Cox model with the outcome being time in weeks for a Federal judge to be confirmed by the Senate from the period 1970 to 2008. This is modeled as a function of a series of variables. The first three are dummy variables for the decade in question. The last is dummy variable for whether one party holds 60 seats in the Senate. Use hazard ratios to interpret the supermajority and 00's variables. What is the key diagnostic for Cox models and how is it remedied?

Table 4: Cox Model of Civil War Duration

00's	1.68
	(0.27)
90's	1.32
	(0.34)
80's	1.03
	(0.43)
Supermajority	−1.23
	(0.40)

PART II. Answer two of the questions below.

1. Explain the concept of the bootstrap. What does the bootstrap simulate? When can the bootstrap fail?

2. Matching has developed as an alternative to regression models for the estimating of causal effects. What assumptions underlie a matched analysis? How do those assumptions compare to the usual regression model assumptions? What role does sensitivity analysis play in an analysis based on matching? Assume you have conducted a sensitivity analysis with $\Gamma = 2$ and the p-value is now .26. What does this imply and how would you interpret this?

3. A classic game used to demonstrate a mixed strategy Nash equilibrium is called Rock-Paper-Scissors. In this game, two players simultaneously select one of the three moves. If both players select the same move, both receive utility of zero. If one selects Rock and the other plays Paper, the player selecting Rock receives utility -1 and the one playing Paper receives +1 (Paper covers Rock). If one selects Rock and the other plays Scissors, the one playing Rock receives +1 and the one playing Scissors receives -1 (Rock crushes Scissors). Finally, if one plays Paper and the other plays Scissors, the one playing Paper receives -1 and the one playing Scissors receives +1 (Scissors cuts Paper). The unique Nash equilibrium to this game is that both players play the mixed strategy 1/3 Rock, 1/3 Paper, 1/3 Scissors.

   In this problem, consider a variant on this classic game. In this variant, if one player plays Rock and the other plays Scissors, the Rock player receives +2 and the Scissors player receives -2 (Rock REALLY crushes Scissors!). The utilities from all other pairs of moves played against one another remain as before.

   (a) Draw the extensive form of this game.

   (b) Construct the normal form of this game.

   (c) Find all Nash equilibria of this game (consider both pure and mixed strategies).

   (d) Show that, for Player 1 playing the mixed strategy equilibrium derived in part (c), Player 2 receives the same expected utility no matter what strategy she plays. What is that expected utility?

   (e) In the mixed strategy Nash equilibrium to this game, is Rock or Paper or Scissors played with the greatest frequency? Why?

4. Explain Krosnick's application of the satisficing notion to the evaluation of survey questions, and also explain Tourangeaus four-component theory of survey response. Would you consider these two approaches to be opposing one another or complementing one another, and why? How can these two perspectives be used to help guide question writing and questionnaire construction?

5. What is a unit root? Why is it important to determine if your time series contains a unit root? What substantive conclusions can you draw from this determination? Discuss

the Dickey-Fuller, Phillips-Peron, and KPSS tests that could be used to analyze the nature of your data. Talk about the advantages/disadvantages of obtaining an estimate of d from fractional integration in terms of assessing whether there is a unit root. If you are interested in multivariate analysis, what are possible next steps after determining the value of d?

6. What errors in survey research can arise from problems with the sampling frame? Describe the different types of error that can occur, and give examples of each.

7. Experimental research is lauded for its superior "internal validity." What does this phrase mean? How is internal validity achieved? What is a true experiment anyway, and how does it differ from other varieties of experimental research (field experiments, quasi-experiments, simulations)? What are some of the major threats to internal validity, and how can they be minimized through careful design? In your answer, compare experimental designs to "pseudoexperiments" and cite specific examples from the experimental literature whenever possible.

8. Find the solution of the difference equation for the given initial condition. Graph the solution sequence with time on the horizontal axis. Characterize the limiting behavior of the sequence.

$$Y_{t+1} - 3Y_t = .10 \;\; Y_0 = 1$$
$$4Y_{t+1} - Y_t = 2 \;\; Y_0 = 4$$
$$Y_{t+1} + Y_t = 0 \;\; Y_0 = -10$$
$$3Y_{t+1} + Y_t = 1 \;\; Y_0 = 1$$
$$7Y_{t+1} + 7Y_t = 3 \;\; Y_0 = 5$$

Why are nonlinear difference equation important in the study of time series analysis?